Cosmology 2 TA: Symmetry restoration: $U(1)$ theory and electroweak theory

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1 Symmetry restoration

You saw in class that the tree level effective potential of the Higgs (or of any field really) gets corrections at one loop. The structure of the potential near the (actually false) minimum we reside in today does not change when applying these corrections at zero temperature, i.e. in an empty background. When we go back in time, on the early universe, the temperature was much higher (today, $T_0 \simeq 2.7$ K), and thermal effects cannot be neglected any more. Thermal effects do change the structure of the effective potential, see Fig. 1. This means that the vacuum state, in the early universe, was indeed at $\chi = 0$. This process is called symmetry restoration, because thermal effects "pushes" the vacuum state back to $\chi = 0$, "undoing" the spontaneous symmetry breaking.

I would like to remark that symmetry restoration is a little of a misnomer. First of all, when discussing spontaneous symmetry breaking, there is no actual symmetry which gets broken: the standard model remains gauge invariant, even when the actual vacuum state does not reside at $\chi = 0$. What gets "broken" is the invariance of the vacuum state itself, not of the theory, hence the word symmetry used in this context needs caution. Also, the word restoration implies that you are restoring something that was broken, but the arrow of time, in what we are gonna talk, points towards the "intact to broken" direction.

Having clarified this, we will now try to discuss the computation of V_{eff} to a simple $U(1)$ gauge theory, and how to extend to the electroweak theory (in a not too rigorous way).

2 A simple $U(1)$ gauge theory

Consider a $U(1)$ gauge theory for a complex scalar field ϕ

$$S = \int \mathrm{d}^4x \left(\frac{1}{2} (\mathrm{D}^\mu \phi)^* D_\mu \phi - V(\phi\phi^*) - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right) =: \int \mathrm{d}^4x \, \mathcal{L} \ , \ D_\mu = \partial_\mu + \mathrm{i}gA_\mu \ , \ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \ ; \tag{2.1}$$

this Lagrangian is invariant under the gauge

$$\phi \to \mathrm{e}^{-\mathrm{i}g\lambda} \phi \ , \ A_\mu \to A_\mu + \partial_\mu \lambda \ . \tag{2.2}$$

It is convenient to define

$$\phi =: \chi \mathrm{e}^{\mathrm{i}g\zeta} \ , \ G_\mu := A_\mu + \partial_\mu \zeta \ , \tag{2.3}$$

where χ and ζ are real fields; notice that χ and G_μ are gauge invariant. These definitions make sense if $\chi \neq 0$. For simplicity, in the following we will always assume this to be the case, or assume that departures from this assumption do not harm our discussion.

With these, the Lagrangian can be written as

$$\mathcal{L} = \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - V(\chi^2) - \frac{1}{4} F_G^2 + g^2 \chi^2 G^\mu G_\mu \ , \tag{2.4}$$

where we see that G_μ can now be interpreted as a massive vector field (hence, we conserve the number of degrees of freedom of the original Lagrangian). The last term is found by expanding the covariant derivative term $(\mathrm{D}^\mu \phi)^* D_\mu \phi$ in terms of the new fields. The equation of motion for χ can be obtained via

$$\frac{\delta S}{\delta \chi} = 0 \implies \frac{\partial \mathcal{L}}{\partial \chi} = \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \chi} \ , \tag{2.5}$$

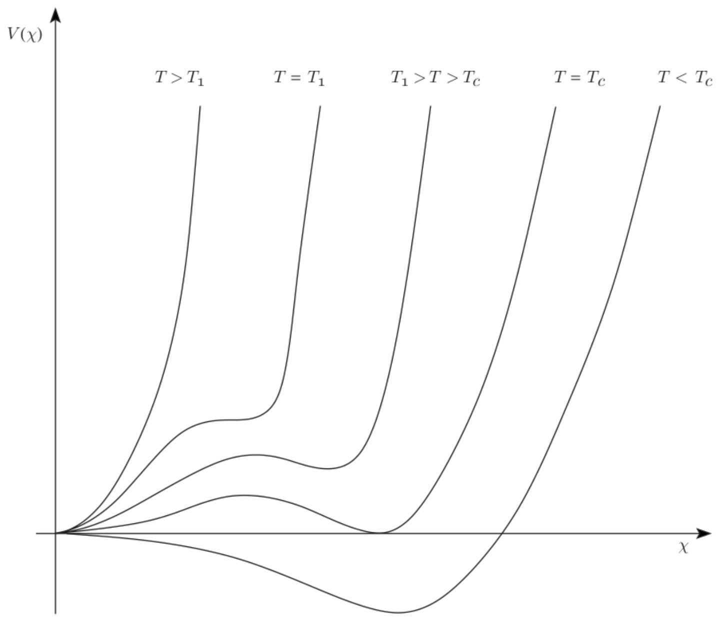

Figure 1: Example of the behavior of the effective potential with temperature. From Mukhanov book.

implying

$$\partial^\mu \partial_\mu \chi + \frac{\partial V}{\partial \chi} - g^2 \chi G^\mu G_\mu = 0 \ . \tag{2.6}$$

Expand $\chi = \chi_c + \varphi$, where φ is the quantum fluctuation, with $\langle \varphi \rangle = 0$. Inserting this split in the previous, we obtain

$$\partial^\mu \partial_\mu \chi_c + \frac{\partial V(\chi_c)}{\partial \chi} + \frac{\varphi^2}{2} \frac{\partial^3 V(\chi_c)}{\partial \chi^3} - g^2 G^2 \chi_c + \left[\partial^\mu \partial_\mu \varphi + \varphi \frac{\partial^2 V(\chi_c)}{\partial \chi^2} - g^2 \varphi G^2 \right] = 0 \ , \tag{2.7}$$

where we expanded V on φ and retained up to second order in φ. When we average the previous, the term in square brackets vanishes due to $\langle \varphi \rangle = 0$, remaining with

$$\partial^\mu \partial_\mu \chi_c + \frac{\partial V(\chi_c)}{\partial \chi} + \frac{\langle \varphi^2 \rangle}{2} \frac{\partial^2 m_\chi^2(\chi_c)}{\partial \chi} - \frac{1}{2} \frac{\partial m_G^2(\chi_c)}{\partial \chi} \langle G^2 \rangle =: \partial^\mu \partial_\mu \chi_c + \frac{\partial (V(\chi_c) + V_{\text{eff}}(\chi_c))}{\partial \chi} \ , \tag{2.8}$$

where we defined $m_G(\chi) := \chi g$, and where the effective potential (vacuum plus thermal) we wish to compute reads

$$\frac{\partial (V_1(\chi_c) + V_T(\chi_c))}{\partial \chi} = \frac{\langle \varphi^2 \rangle}{2} \frac{\partial^3 V(\chi_c)}{\partial \chi^3} - g^2 \chi_c \langle G^2 \rangle \ . \tag{2.9}$$

The equation of motion for G^μ reads

$$\partial_\nu \partial^\nu G_\mu - \partial_\mu \partial^\nu G_\nu + g^2 \chi^2 G_\mu = 0 = \partial_\nu \partial^\nu G_\mu + g^2 \chi^2 G_\mu \ , \tag{2.10}$$

where we used $\partial^\nu G_\nu = 0$ (just take the ∂_μ derivative to see this). In particular,

$$\frac{\partial V(G)}{\partial G^\mu} = g^2 \chi^2 G_\mu \ , \tag{2.11}$$

from which one can infer $\langle G_\mu \rangle = 0$ due to the form of the potential, hence $\langle G^2 \rangle$ is the quantum fluctuations part already.

We can compute $\langle G^2 \rangle$ very similarly as how we computed $\langle \varphi^2 \rangle$, as we will see in the next section.

3 Thermal effects

Consider a field component ψ_i, where ψ_i is real; a complex scalar field would have 2 components, a massive vector field would have 3, a left-handed or right-handed fermion would have 2. In the following, we assume we deal with a scalar, and we will later explain which difference arises in the vector and fermion case. In all this discussion, we neglected minimal couplings with the metric; this assumes that the metric dynamics is slow and locally negligible. If this holds, we are allowed to locally expand

$$\psi_i(x) = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \mathrm{e}^{-\mathrm{i}kx} \hat{\psi}_i(k) \ , \tag{3.1}$$

where $k = (k^0, \vec{k})$ and $kx = k^0 x^0 - \vec{k} \cdot \vec{x}$. Field components equation of motion of any field (when neglecting interactions) are just Klein-Gordon, which, when applied to the previous, implies

$$\psi_i(x) = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \mathrm{e}^{-\mathrm{i}kx} \hat{\psi}_i(k)(-(k^0)^2 + \vec{k} + m_\varphi^2) = 0 \ ; \tag{3.2}$$

the previous is always satisfied if $\hat{\psi}_i(k) = 0$ when $-(k^0)^2 + \vec{k} + m_\varphi^2 \neq 0$. This means we can write

$$\psi_i(k) = \delta(-(k^0)^2 + \omega_k^2) f_i(\vec{k}) \ , \ \omega_k := \sqrt{\vec{k} + m_{\psi_i}^2} \ ; \tag{3.3}$$

exploiting the relation

$$\delta(g(x)) = \sum_i \frac{\delta(x - x_{0i})}{|g'(x_{0i})|} \ , \tag{3.4}$$

where x_{0i} are the zeros of the function g, we have (in our case, $x_{0i} = \pm\omega_k$, $g(k^0) = \sqrt{-(k^0)^2 + \omega_k^2}$)

$$\psi_i(x) = \int \frac{\mathrm{d}^3 k}{(2\pi)^4 2\omega_k} \Big(f(\vec{k}) \ \mathrm{e}^{-\mathrm{i}kx}\Big|_{k^0=\omega_k} + f(\vec{k}) \ \mathrm{e}^{-\mathrm{i}kx}\Big|_{k^0=-\omega_k} \Big) \ ; \tag{3.5}$$

defining

$$a_i(\vec{k}) = (2\pi)^{5/2} \sqrt{2\omega_k} f_i(\vec{k}) = a^*_{-\vec{k}} \ , \tag{3.6}$$

where the last equality comes from the fact that ψ_i is real, we can write

$$\psi_i(x) = \int \frac{\mathrm{d}^3 k}{(2\pi)^{3/2} \sqrt{2\omega_k}} \Big(a_i(\vec{k}) \mathrm{e}^{-\mathrm{i}kx} + a_i^\dagger(\vec{k}) \ \mathrm{e}^{\mathrm{i}kx} \Big)\Big|_{k^0=\omega_k} \ , \tag{3.7}$$

where we promoted $a_{\vec{k}}$ to an operator. The normalization we used in Eq. (3.6) ensures that the equal time commutation relation

$$[\psi_i(x), \partial_0 \psi(y) \ ]\Big|_{x^0=y^0} = \mathrm{i}\delta(\vec{x} - \vec{y}) \ , \tag{3.8}$$

translates into

$$[a_i(\vec{k}), a_i^\dagger(\vec{k}')] = \delta(\vec{k} - \vec{k}') \ . \tag{3.9}$$

Recall to change Eq. (3.8) and Eq. (3.9) with anticommutators in the case of fermions.

There is a slight difference for the vector (fermion) fields, which come from the polarization vectors ϵ_i^μ (spinors u_i, v_i). The expansion Eq. (3.7) for G^μ reads

$$G^\mu(x) = \int \frac{\mathrm{d}^3 k}{(2\pi)^{3/2} \sqrt{2\omega_k}} \Big(\epsilon_i^\mu a_i(\vec{k}) \mathrm{e}^{-\mathrm{i}kx} + \epsilon_i^{*\mu} a_i^\dagger(\vec{k}) \ \mathrm{e}^{\mathrm{i}kx} \Big)\Big|_{k^0=\omega_k} \ , \tag{3.10}$$

whereas for (Dirac) fermions

$$\Psi(x) = \sum_{i=1,2} \int \frac{\mathrm{d}^3 k}{(2\pi)^{3/2} \sqrt{2\omega_k}} \Big(u_i a_i(\vec{k}) \mathrm{e}^{-\mathrm{i}kx} + v_i b_i^\dagger(\vec{k}) \ \mathrm{e}^{\mathrm{i}kx} \Big)\Big|_{k^0=\omega_k} \ . \tag{3.11}$$

Suppose now that we have a grand-canonical ensemble of ψ_i particles at temperature T. It is characterized by

$$\rho = \frac{\mathrm{e}^{-\beta(H-\mu N)}}{\mathrm{Tr}\big(\mathrm{e}^{-\beta(H-\mu N)}\big)} \ , \tag{3.12}$$

where H is the Hamiltonian of the system, $\beta = 1/T$ and the trace is over all the (orthonormal) states of the system, N is the number operator and μ the chemical potential. The expectation value of an operator A would read

$$A = \mathrm{Tr}(\rho A) \ . \tag{3.13}$$

Recall that we can build the Fock space of particles with wavevector $\vec{k}$ via

$$\left|n_{\vec{k}}\right\rangle \propto \frac{(a_i^\dagger(\vec{k}))^n}{\sqrt{n!}}\left|0\right\rangle \ . \tag{3.14}$$

With this, we can write the number operator

$$N = \int \mathrm{d}^3k\, a_i^\dagger(\vec{k}) a_i(\vec{k}) \implies N\left|n_{\vec{k}}\right\rangle = n_{\vec{k}}\left|n_{\vec{k}}\right\rangle \ , \tag{3.15}$$

and the Hamiltonian would read

$$H = \int \mathrm{d}^3k\, \omega_k a_i^\dagger(\vec{k}) a_i(\vec{k}) \implies H\left|n_{\vec{k}}\right\rangle = \omega_k n_{\vec{k}}\left|n_{\vec{k}}\right\rangle \ . \tag{3.16}$$

Any possible state can be characterized with the set $\{n_{\vec{k}}\} := \{0_{\vec{k}}, 1_{\vec{k}}, \ldots\}$, where a single $n_{\vec{k}}$ denotes the number n of particles with wavevector $\vec{k}$. Then

$$Z := \mathrm{Tr}\left(\mathrm{e}^{-\beta(H-\mu N)}\right) = \sum_{\{n_{\vec{k}}\}} \left\langle \{n_{\vec{k}}\}\middle|\mathrm{e}^{-\beta(H-\mu N)}\middle|\{n_{\vec{k}}\}\right\rangle = \sum_{\{n_{\vec{k}}\}} \mathrm{e}^{-\beta\sum_{\vec{k}} n_{\vec{k}}(\omega_k-\mu)} = \prod_{\vec{k}}\sum_{n=0}^{\alpha} \mathrm{e}^{-\beta n(\omega_k-\mu)} \ , \tag{3.17}$$

where $\alpha = \infty, 1$ for bosons and fermions respectively (fermions cannot occupy the same $\vec{k}$ state). Using the geometric series for the boson case, and simple two terms sum for fermions, we can write

$$Z = \begin{cases} \prod_{\vec{k}} \frac{1}{1-\mathrm{e}^{-\beta(\omega_k-\mu)}} & \text{for bosons ,} \\ \prod_{\vec{k}}(1+\mathrm{e}^{-\beta(\omega_k-\mu)}) & \text{for fermions ;} \end{cases} \tag{3.18}$$

Then one can easily compute

$$\langle N\rangle = \frac{1}{\beta}\frac{\partial \ln Z}{\partial\mu} = \sum_{\vec{k}} f_{\vec{k}} := \begin{cases} \sum_{\vec{k}} \frac{1}{\mathrm{e}^{\beta(\omega_k-\mu)}-1} & \text{for bosons ,} \\ \sum_{\vec{k}} \frac{1}{\mathrm{e}^{\beta(\omega_k-\mu)}+1} & \text{for fermions ;} \end{cases} \tag{3.19}$$

from which one can infer[1]

$$\langle a_i^\dagger(\vec{k}) a_i(\vec{k}')\rangle = \delta(\vec{k}-\vec{k}')\frac{1}{\mathrm{e}^{\beta(\omega_k-\mu)}\pm 1} = \delta(\vec{k}-\vec{k}') f_{\vec{k}} \ . \tag{3.23}$$

We are finally ready to compute the expectation value (recall $\langle a_{\vec{k}} a_{\vec{k}'}\rangle = \langle a^\dagger_{\vec{k}} a^\dagger_{\vec{k}'}\rangle = 0$)

$$\begin{aligned} \langle\psi_i^2\rangle &= \int \frac{\mathrm{d}^3k}{(2\pi)^{3/2}}\frac{\mathrm{d}^3k'}{(2\pi)^{3/2}2\sqrt{\omega_k\omega_{k'}}}\Big(\langle a_i^\dagger(\vec{k}) a_i(\vec{k}')\rangle \mathrm{e}^{-\mathrm{i}x(\vec{k}'-\vec{k})} + \underbrace{\langle a_i(\vec{k}) a_i^\dagger(\vec{k}')\rangle}_{\langle a_i^\dagger(\vec{k}') a_i(\vec{k})\rangle+\delta(\vec{k}-\vec{k}')} \mathrm{e}^{\mathrm{i}x(\vec{k}'-\vec{k})}\Big) \\ &= \int \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{1}{\omega_k}\left(\frac{1}{2}+f_{\vec{k}}\right) \ . \end{aligned} \tag{3.24}$$

This results holds for bosons. Notice that, for $\langle G^\mu G_\mu\rangle$, taking into account the polarization ϵ_i^μ, we would have, exploiting the relation $\epsilon_i^\mu\epsilon_{j\mu} = -\delta_{ij}$

$$G^\mu G_\mu \sim (\epsilon_i^\mu a_i + \epsilon_i^{\mu *} a_i^\dagger)(\epsilon_{j\mu} a_j + \epsilon_{j\mu}^* a_j^\dagger) = -\sum_{i=1}^{3}(a_i + a_i^\dagger)(a_i + a_i^\dagger) \tag{3.25}$$

[1] Notice

$$\langle n_{\vec{k}''}|\mathrm{e}^{-\beta(H-\mu N)} a^\dagger_{\vec{k}'} a_{\vec{k}}|n_{\vec{k}''}\rangle = \langle n_{\vec{k}''}|\mathrm{e}^{-\beta n_k(\omega_k-\mu)}\delta(\vec{k}''-\vec{k}) n_k a^\dagger_{\vec{k}'}|(n-1)_{\vec{k}''}\rangle = \begin{cases} 0 & , \vec{k}'' \neq \vec{k}' \\ n_k \mathrm{e}^{-\beta n_k(\omega_k-\mu)}\delta(\vec{k}'-\vec{k}) & , \vec{k}'' = \vec{k}' \end{cases} \tag{3.20}$$

So

$$\langle a_i^\dagger(\vec{k}) a_i(\vec{k}')\rangle = \delta(\vec{k}'-\vec{k})\frac{\sum_{n=0}^{\alpha}\mathrm{e}^{-\beta n(\omega_k-\mu)}}{\prod_{\vec{k}''}\sum_{n=0}^{\alpha}\mathrm{e}^{-\beta n(\omega_{k''}-\mu)}} \ , \tag{3.21}$$

to be compared with

$$\frac{1}{\beta}\frac{\partial\ln Z}{\partial\mu} = \sum_{\vec{k}}\frac{\sum_{n=0}^{\alpha}\mathrm{e}^{-\beta n(\omega_k-\mu)}}{\prod_{\vec{k}''}\sum_{n=0}^{\alpha}\mathrm{e}^{-\beta n(\omega_{k''}-\mu)}} \ . \tag{3.22}$$

which would yield -3 times the result for a single scalar field. Notice that 3 is the number of degrees of freedom of a massive vector field. Generically, for a vector field, we can write

$$\langle G^\mu G_\mu \rangle /(\text{d.o.f}) = -\int \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{1}{\omega_k} \left(\frac{1}{2} + f_{\vec{k}} \right) . \tag{3.26}$$

For fermions, instead, we would have to insert the spinors $u_i, v_i,\ i = 1, 2$; Exploiting $\sum_i \bar{u}_i u_i = 4m = -\sum_i \bar{v}_i v_i$, we have (schematically)

$$\langle \bar{\Psi}\Psi \rangle \sim \left\langle (\bar{v}_j b_j + \bar{u}_j a_j^\dagger)(u_i a_i + v_i b_i^\dagger) \right\rangle = \sum_i \bar{u}_i u_i \langle a_i^\dagger a_i \rangle + \sum_i \bar{v}_i v_i \langle b_i b_i^\dagger \rangle = 4m \langle a_1^\dagger a_1 \rangle - 4m(-\langle b_1^\dagger b_1 \rangle + \delta(\vec{k} - \vec{k}')) \tag{3.27}$$

where we accounted for anticommutation rules for fermions; we thus have, per degree of freedom

$$\langle \bar{\Psi}\Psi \rangle /\text{d.o.f.} = m_{\psi_i} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{1}{\omega_k} \left(-\frac{1}{2} + f_{\vec{k}} \right) . \tag{3.28}$$

The vacuum contribution can be written

$$\frac{\partial V_1(\chi_c)}{\partial \chi} = \frac{1}{2} \frac{\partial m^2_{\psi_i}(\chi_c)}{\partial \chi} \frac{1}{2} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{1}{\sqrt{\vec{k}^2 + m^2_{\psi_i}(\chi_c)}} = \left. \frac{\partial}{\partial \chi} \right|_{\chi = \chi_c} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{\sqrt{\vec{k}^2 + m^2_\psi(\chi)}}{2} =: \frac{I(m^2_{\psi_i})}{4\pi^2} \tag{3.29}$$

(just do the derivative to see that this holds). Regularizing the divergence and renormalizing at a scale μ, we can rewrite it

$$V_1 = \frac{m^4_{\psi_i}(\chi_c)}{64\pi^2} \ln \frac{m^2_{\psi_i}(\chi_c)}{\mu^2} . \tag{3.30}$$

The thermal contribution can be written as

$$\begin{aligned} V_T &= \frac{1}{2} \frac{\partial^2 m^2(\chi_c)}{\partial \chi} \int \frac{\mathrm{d}k}{(2\pi)^3} \frac{4\pi k^2}{\sqrt{k^2 + m^2_{\psi_i}(\chi_c)}} \frac{1}{\mathrm{e}^{\beta\sqrt{k^2 + m^2_{\psi_i}(\chi_c)}} \pm 1} \\ &= \frac{1}{4\pi^2 \beta^4} \frac{\partial^2 \beta^2 m^2(\chi_c)}{\partial \chi} \int \frac{\mathrm{d}x\, x^2}{\sqrt{x^2 + \beta^2 m^2_{\psi_i}(\chi_c)}} \frac{1}{\mathrm{e}^{\sqrt{x^2 + \beta^2 m^2_{\psi_i}(\chi_c)}} \pm 1} \\ &= \mp \frac{1}{2\pi^2} T^4 \left. \frac{\partial}{\partial \chi} \right|_{\chi = \chi_c} \int \mathrm{d}x\, x^2 \ln \left(1 \pm \mathrm{e}^{-\sqrt{x^2 + \beta^2 m^2_{\psi_i}(\chi)}} \right) =: \frac{1}{4\pi^2} F_\pm \left(\frac{m_{\psi_i}}{T} \right) . \end{aligned} \tag{3.31}$$

Notice that the same results hold for vector and fermions; one should notice that the couplings of fermions and vectors to scalars follow

$$\mathcal{L} \sim -m_G^2 G^\mu G_\mu + \frac{m_\Psi}{\chi_c} \bar{\Psi}\Psi = \frac{1}{2} \frac{\partial m_G^2(\chi_c)}{\partial \chi} (-G^\mu G_\mu) + \frac{1}{2} \frac{\partial m_\Psi^2(\chi_c)}{\partial \chi} \left(\frac{1}{m} \bar{\Psi}\Psi \right) \tag{3.32}$$

where $m_\Psi(\chi) = y_\Psi \chi$, with y_Ψ the Yukawa coupling. Combining the previous with Eq. (3.26) and Eq. (3.28), we can conclude with the general result

$$V_{\text{eff}} = \sum_{\text{bosons}} \left(g_i \frac{m^4_{\psi_i}(\chi_c)}{64\pi^2} \ln \frac{m^2_{\psi_i}(\chi_c)}{\mu^2} + \frac{g_i}{4\pi^2} F_\pm \left(\frac{m_{\psi_i}}{T} \right) \right) + \sum_{\text{fermions}} \left(-g_i \frac{m^4_{\psi_i}(\chi_c)}{64\pi^2} \ln \frac{m^2_{\psi_i}(\chi_c)}{\mu^2} + \frac{g_i}{4\pi^2} F_\pm \left(\frac{m_{\psi_i}}{T} \right) \right) \tag{3.33}$$

where g_i is the number of degrees of freedom for boson/fermion i.

4 Back to $U(1)$ gauge theory and extension to electroweak theory

Armed with Eq. (3.33), we can write

$$\langle G^2 \rangle = \frac{\partial}{\partial \chi} \left(\frac{3}{4\pi^2} \left(I(m_G) + T^4 F_-(m_G/T) \right) \right) , \tag{4.1}$$

where $m_G = \chi_c g$, and where the factor 3 comes from the 3 degrees of freedom of a massive vector field. Hence,

$$V_{\text{eff}} = V(\chi_c) + \frac{3m_G^4(\chi_c)}{64\pi^2} \ln \frac{m_G^2(\chi_c)}{\mu^2} + \frac{3T^4 F_-(m_G/T)}{4\pi^2} \ . \tag{4.2}$$

We can extend the previous to the Higgs in electroweak theory; considering the equation of motion for an Higgs-like field

$$\partial_\mu \partial^\mu \chi_c + V'(\chi_c) - \frac{g^2 + g'^2}{4} \chi_c \langle Z_\mu Z^\mu \rangle - \frac{g^2}{2} \chi_c \left\langle W_\mu^+ W^{-\mu} \right\rangle + \frac{m_t}{\chi_c} \langle \bar{t} t \rangle = 0 \ , \tag{4.3}$$

where we included the Yukawa coupling for the top quark (there should be the one for all fermions, but the top is the one that dominates), and we have the contribution from the 3 vector bosons Z^μ, W_μ^+, W_μ^-. Then, summing all the different contribution of the particles, with their number of degrees of freedom factor, we obtain the V_{eff} coming from all other non-Higgs particles to be

$$V_{\text{eff}} = V(\chi_c) + \frac{1}{64\pi^2} \left(3m_Z^4 \ln \frac{m_Z^2}{\mu^2} + 6m_W^4 \ln \frac{m_W^2}{\mu^2} - 12 m_t^2 \ln \frac{m_t^2}{\mu^2} \right) + \frac{1}{4\pi^2} \left(3F_- \left(\frac{m_Z}{T} \right) + 6F_- \left(\frac{m_W}{T} \right) + 12 F_+ \left(\frac{m_t}{T} \right) \right) \tag{4.4}$$

where for the top we counted 3 for the colors, 2 for left and right handed, 2 for the spin.