Weizmann Institute

Comments on the Mass Sheet Degeneracy in Cosmography Analyses

IPS 2022

Teodori Luca Based on LT, K. Blum, E. Castorina, M. Simonović and Y. Soreq (2022) [2201.05111]

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Measuring H_0 with lensing





- TDCOSMO collaboration (COSMOGRAIL, HOLICOW, STRIDES, SHARP, COSMICLENS)
- Unaccounted galactic density profile features (core?): internal mass sheet degeneracy
- Large scale structure "on the way": external mass sheet degeneracy (weak lensing)
- Can multiple source systems help?

credit: NASA



Strong Gravitational Lensing in Elliptical Galaxies



- $\hat{\alpha} = 2 \int \nabla_{\perp} \Phi \, d\lambda \implies \vec{\beta} = \vec{\theta} \frac{\mathsf{D}_{\mathrm{LS}}}{\mathsf{D}_{\mathrm{S}}} \hat{\alpha}(\vec{\theta})$
- Lens model + time delay measurement

$$\Delta t_{ij} \propto rac{1}{H_0}$$

Degeneracies: source position and mass of galaxy unknown

$$\vec{\beta} \to \lambda \vec{\beta} \; , \; \vec{\alpha} \to \lambda \vec{\alpha} + (1 - \lambda) \implies H_0 \to \lambda H_0$$



from V. Bonvin et al (2016)

Effects of Large Scale Structure



External convergences κ are unobservable, can be removed from modeling (not completely true in multiple source systems!)

Bias in time delays:

$$\frac{H_0^{\text{inferred}}}{H_0^{\text{true}}} = \frac{1 - \kappa^{\text{LS}}}{1 - \kappa^{\text{L}}}$$

Stellar kinematics can constrain mass in the galaxy. Bias:

$$\frac{H_0^{\rm inferred}}{H_0^{\rm true}} \approx 1 + \kappa^{\rm L}$$

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• Which direction? Difficult to say without ray-tracing

Multi-source systems and MSD

Cluster MACS J1149.5+2223. Credit: NASA



Shajib et al 2019





• Large unaccounted cores have similar effects to weak lensing $\kappa^{\rm S}$ (internal mass sheet degeneracy), but it can have larger effects

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$$1 - \lambda = \kappa_{c1} \implies \kappa_{c1} \simeq \frac{\delta H_0}{H_0}$$

• Observable (C_i involves ratio of angular diameter distances):

$$\delta \kappa_{1i}^{\mathrm{S}} - \delta \kappa_{1i}^{\mathrm{LS}} + \frac{\kappa_{\mathrm{c1}}}{(1 - C_{\mathrm{i}})}$$



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External convergence reinterpretation?

DESJ0408-5354



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- With $\delta \kappa^{\text{ext}} := \delta \kappa_{21}^{\text{S}} \delta \kappa_{21}^{\text{LS}}$, $\delta \kappa^{\text{ext}} \rightarrow \delta \kappa^{\text{ext},\lambda} = \delta \kappa^{\text{ext}} + \kappa_{c1} (\mathsf{C}_2 - 1)$
- "Good systems" we have now cannot say much; clusters have still a lot of modeling uncertainties.
- With future surveys, many more multisource systems ⇒ we could spot the core!



- Challenge: modeling degeneracies; important to point out all the shortcomings to lensing collaborations, in other to achieve an *accurate* H₀ measurement.
- Promising! New data arriving, including precise stellar kinematics, new lensed systems etc.
- Effects from observer-lens and lens-source sightlines *must* be taken into account for a non-biased inference of time delays and non-biased stellar kinematics constraint.
- With multi-source systems, you might spot the core (still not possible yet).
- With an H₀ prior, we can measure galactic features like large cores, difficult to spot otherwise, and a better characterization of the weak lensing field

The multilens equation

The tidal approximation



Lens equation in tidal approximation

$$\vec{\beta} = (\mathbb{I} - M^{\mathrm{S}})\vec{\theta} - (\mathbb{I} - M^{\mathrm{LS}})\vec{\alpha}((\mathbb{I} - M^{\mathrm{L}})\vec{\theta})$$

Degeneracy ("revised" MSD)

$$\begin{split} 1 &- \mathsf{M}^{\mathrm{R}} \longmapsto \lambda_{\mathrm{R}} (1 - \mathsf{M}^{\mathrm{R}}), \\ & \vec{\beta} \longmapsto \lambda_{\mathrm{S}} \vec{\beta}, \\ & \vec{\alpha}(\vec{\theta}) \longmapsto \lambda_{\mathrm{S}} \lambda_{\mathrm{LS}}^{-1} \vec{\alpha} (\lambda_{\mathrm{L}}^{-1} \vec{\theta}), \\ & \Psi(\vec{\theta}) \longmapsto \lambda_{\mathrm{S}} \lambda_{\mathrm{LS}}^{-1} \lambda_{\mathrm{L}} \Psi (\lambda_{\mathrm{L}}^{-1} \vec{\theta}) \end{split}$$

Choose

$$\lambda_{\rm S} = \frac{1}{1-\kappa^{\rm S}}, \quad \lambda_{\rm LS} \, = \, \frac{1}{1-\kappa^{\rm LS}}, \quad \lambda_{\rm L} \, = \, \frac{1}{1-\kappa^{\rm L}},$$

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external convergence removed from the modeling

Interpreting the Mass Sheet Degeneracy

Springer et al 2005



Suyu et al 2010





- Change λ_S : changing one's mind about the true κ^S
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$$\kappa^{\rm R} \longmapsto \lambda_{\rm R} \kappa^{\rm R} + (1 - \lambda_{\rm R})$$
$$\Gamma^{\rm R} \longmapsto \lambda_{\rm R} \Gamma^{\rm R}$$

Time delays do change!

$$\begin{split} \Delta \tau &\to \lambda_{\rm S} \lambda_{\rm LS}^{-1} \lambda_{\rm L} \Delta \tau \\ {\rm H}_0 &\to \lambda_{\rm S} \lambda_{\rm LS}^{-1} \lambda_{\rm L} {\rm H}_0 \end{split}$$

- Estimate $\kappa^{\rm S}$ via ray-tracing through Millennium Simulation and characterization of the lens field
- Degeneracy is limited by priors on weak lensing quantities and constraints on mass of lens galaxy (stellar kinematics)

Multi-source systems and MSD

Cluster MACS J1149.5+2223. Credit: NASA



Shajib et al 2019





New lens equation (ignoring lens-lens coupling)

$$\begin{split} \vec{\beta_i} &= (\mathbb{I} - M_i^{\mathrm{S}}) \vec{\theta} - (\mathbb{I} - M_i^{\mathrm{LS}}) C_i \vec{\alpha}_1 ((\mathbb{I} - M^{\mathrm{L}}) \vec{\theta}) \\ C_i &:= \frac{D_{\mathrm{S}_1} D_{\mathrm{LS}_1}}{D_{\mathrm{LS}_1} D_{\mathrm{S}_i}} , \left(\hat{\alpha} \frac{D_{\mathrm{LS}}}{D_{\mathrm{S}}} =: \vec{\alpha} \right) \end{split}$$

- Observable:

$$C_{i} \frac{\left|\vec{\alpha}_{1}^{\mathrm{model}}(\vec{\theta})\right|}{\left|\vec{\alpha}_{2}^{\mathrm{model}}(\vec{\theta})\right|} \approx \left(1 + \delta \kappa_{12}^{\mathrm{S}} - \delta \kappa_{12}^{\mathrm{LS}}\right)$$